

Trig Derivatives Day 2

I. Trig Derivatives

$$A. \frac{d}{dx} [\sin x] = \cos x \cdot x'$$

$$B. \frac{d}{dx} [\cos x] = -\sin x \cdot x'$$

$$C. \frac{d}{dx} [\tan x] = \sec^2 x \cdot x'$$

$$D. \frac{d}{dx} [\cot x] = -\csc^2 x \cdot x'$$

$$E. \frac{d}{dx} [\sec x] = \sec x \tan x \cdot x'$$

$$F. \frac{d}{dx} [\csc x] = -\csc x \cot x \cdot x'$$

II. Natural Log Derivatives

$$A. \frac{d}{dx} [\ln x] = \frac{x'}{x}$$

$$B. \frac{d}{dx} [e^x] = e^x \cdot x'$$

III. Day 2 Examples:

**With trig functions A – C, the Chain Rule means that your derivative will be found by:
(Derivative of trig function)(Derivative of Angle)

$$A. f(x) = x + \sec(x^2 + \sqrt{2})$$

$$f'(x) = 1 + \sec(x^2 + \sqrt{2}) \tan(x^2 + \sqrt{2}) \cdot (2x)$$

$$f'(x) = 1 + 2x \sec(x^2 + \sqrt{2}) \tan(x^2 + \sqrt{2})$$

$$B. y = -\csc(x^2 + 7x)$$

$$f'(x) = -[-\csc(x^2 + 7x) \cot(x^2 + 7x)] \cdot (2x + 7)$$

$$f'(x) = (2x + 7) \csc(x^2 + 7x) \cot(x^2 + 7x)$$

C. $f(x) = 5 \cot\left(\frac{2}{x}\right)$ rewrite as $f(x) = 5 \cot(2x^{-1})$

$$f'(x) = -5 \csc^2(2x^{-1}) \cdot (-2x^{-2})$$

Apply derivative for $\cot x$

$$f'(x) = 10x^{-2} \csc^2(2x^{-1})$$

Multiply coefficients

$$f'(x) = \frac{10}{x^2} \csc^2\left(\frac{2}{x}\right)$$

Write with no negative exponents

D. $y = (\csc x + \cot x)^{-1}$ recognize as Chain Rule

$$y' = -1(\csc x + \cot x)^{-2} \cdot [-\csc x \cot x - \csc^2 x]$$

$$y' = (\csc x + \cot x)^{-2} \cdot [\csc x \cot x + \csc^2 x]$$

Distribute -1

$$y' = \frac{\csc x \cot x + \csc^2 x}{(\csc x + \cot x)^2}$$

Write with no negative exponents

$$y' = \frac{\csc x(\cot x + \csc x)}{(\csc x + \cot x)^2}$$

Factor the numerator

$$y' = \frac{\csc x}{(\csc x + \cot x)}$$

Reduce like factors

E. $y = \sec^3(3x)$ recognize as Chain Rule (Power Rule)(Derivative of trig function)(Derivative of angle)

$$y' = 3 \sec^2(3x) [\sec(3x) \tan(3x)] \cdot (3)$$

Apply Chain Rule

$$y' = 9 \sec^3(3x) \tan(3x)$$

Clean up by multiplying coefficients & like trig functions

F. $y = \cos^4(5x)$ recognize as Chain Rule (Power Rule)(Derivative of trig function)(Derivative of angle)

$$y' = 4 \cos^3(5x) [-\sin(5x)] \cdot (5)$$

Apply Chain Rule

$$y' = -20 \cos^3(5x) \sin(5x)$$

Multiply coefficients

G. $f(x) = \ln(3x)$

$$f'(x) = \frac{3}{3x}$$

Apply natural log rule

$$f'(x) = \frac{1}{x}$$

Reduce

$$\text{H. } f(x) = \ln(5x^2)$$

$$f'(x) = \frac{10x}{5x^2}$$

Apply natural log rule

$$f'(x) = \frac{2}{x}$$

Reduce

$$\text{I. } y = \ln^3(4x) \quad \text{recognize as Chain Rule}$$

$$y' = 3 \ln^2(x) \cdot \left[\frac{4}{4x} \right]$$

Apply natural log rule

$$y' = \frac{12 \ln^2(x)}{4x}$$

Multiply

$$y' = \frac{3 \ln^2(x)}{x}$$

Reduce

$$\text{J. } f(x) = e^{8x^2+4x}$$

$$f'(x) = e^{8x^2+4x} \cdot (16x+4)$$

Apply natural log rule

$$f'(x) = (16x+4)e^{8x^2+4x}$$

Clean up