4/26: Derivatives of Trigonometric Functions

I. Trig Derivatives

A. $\frac{d}{dx}[\sin x] = \cos x \cdot x'$

B. $\frac{d}{dx}[\cos x] = -\sin x \cdot x'$

C. $\frac{d}{dx}[\tan x] = sec^2 x \cdot x'$

D. $\frac{d}{dx}[\cot x] = -csc^2 x \cdot x'$

E. $\frac{d}{dx}[\sec x] = \sec x \tan x \cdot x'$

F. $\frac{d}{dx}[\csc x] = -\csc x \cot x \cdot x'$

II. Natural Log Derivatives

A. $\frac{d}{dx}[\ln x] = \frac{x'}{x}$

B. $\frac{d}{dx}[e^x] = e^x \cdot x'$

II. Day 1 Examples: Find the derivative

With trig functions A – F, the Chain Rule means that your derivative will be found by:
(Derivative of trig function)(Derivative of Angle)

A. $f(x) = \sin(2x)$

$f'(x) = \cos(2x) \cdot 2$

$f'(x) = 2\cos(2x)$

B. $f(x) = \cos(3x)$

$f'(x) = -\sin(3x) \cdot 3$

$f'(x) = -3\sin(3x)$

C. $f(x) = 5\cos(7x)$

$f'(x) = 5[-\sin(7x) \cdot 7]$

$f'(x) = -35\sin(7x)$

D. $f(x) = 2\sin(4x) - 3\tan(2x)$

$f'(x) = 2\cos(4x) \cdot 4 - 3sec^2(2x) \cdot 2$

$f'(x) = 8\cos(4x) - 6sec^2(2x)$

E. $f(x) = \sin(x^2 + x)$

$f'(x) = \cos(x^2 + x) \cdot (2x + 1)$

$f'(x) = (2x + 1)\cos(x^2 + x)$

F. $f(x) = \tan(\sqrt{x}) \quad \rightarrow \tan\left(x^{\frac{1}{2}}\right)$

$f'(x) = sec^2\left(x^{\frac{1}{2}}\right) \cdot \left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$f'(x) = \frac{sec^2\left(x^{\frac{1}{2}}\right)}{2x^{-\frac{1}{2}}}$

G. $f(x) = x^3\sin(2x)$ recognize this as the product rule where $f(x) = x^3$ $and$ $g(x) = \sin(2x)$

$f'(x) = x^3[\cos(2x) \cdot 2] + \sin(2x) \cdot 3x^2$ Apply product rule

$f'(x) = 2x^3\cos(2x) + 3x^2\sin(2x)$ Clean up by moving coefficients to the front of each term

H. $f(x) = \sin(3x)\cos(5x)$ recognize this as the product rule where $f(x) = \sin(3x)$ $and$ $g(x) = \cos(5x)$

$f'(x) = \sin(3x)[-\sin(5x) \cdot 5] + \cos(5x)[\cos(3x) \cdot 3]$ Apply product rule

$f'(x) = -5\sin(3x)\sin(5x) + 3\cos(5x)\cos(3x)$ Clean up by moving coefficients to the front

**You cannot combine the functions of the sine/cos together because they are different functions

I. $f(x) = \cos^3(2x)$ recognize this as the chain rule think of the function as $[\cos(2x)]^3$

$f'(x) = 3cos^2(2x) \cdot [-\sin(2x) \cdot 2]$ Apply chain rule

$f'(x) = -6cos^2(2x)\sin(2x)$ Clean up by moving coefficients to the front

J. $\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$ recognize this as the quotient rule

$= \frac{\cos x\ (\cos x) - \sin x\ (\sin x)}{cos^2 x}$ Apply quotient rule

$= \frac{cos^2 x + sin^2 x}{cos^2 x}$ Multiply in the numerator

$= \frac{1}{\cos^2 x}$ Trig Identity in the numerator

$= \sec^2 x$ Reciprocal Identity

J. $f(x) = \frac{\cos x}{1 - \sin x}$ recognize the quotient rule

$f'(x) = \frac{-\sin x\ (1 - \sin x) - \cos x\ (-\cos x)}{(1 - \sin x)^2}$ Apply quotient rule

$f'(x) = \frac{-\sin x + sin^2 x + cos^2 x}{(1 - \sin x)^2}$ Multiply in the numerator

$f'(x) = \frac{-\sin x + 1}{(1 - \sin x)^2}$ Trig Identity in the numerator

$f'(x) = \frac{1}{(1 - \sin x)}$ Cancel out b/c like values

K. $\frac{d}{dx}[e^{2x} \tan x]$ recognize as the product rule with $f(x) = e^{2x}$ *and* $g(x) = \tan x$

$= e^{2x}\ [sec^2 x] + \tan x\ [e^{2x} \cdot 2]$ Apply product rule

$= e^{2x}\ sec^2 x + 2e^{2x} \tan x$ Move coefficients

$= e^{2x}(sec^2 x + 2 \tan x)$ Factor