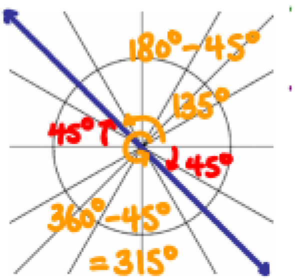
Solving Trig Equations

Example 1: Determine the exact measure of all angles that satisfy the given conditions.

a) $\tan\theta = -1 \qquad 0° \le \theta < 360°$

Tangent is negative so we are in Q2 or Q4
The reference angle is 45°

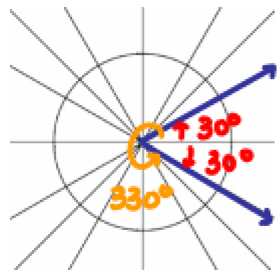
$\theta_1 = 135°\ \ and\ \ \theta_2 = 315°$



b) $\cos\theta = \frac{\sqrt{3}}{2} \qquad 0° \le \theta < 360°$

Cosine is positive in Q1 or Q4
The reference angle is 30°

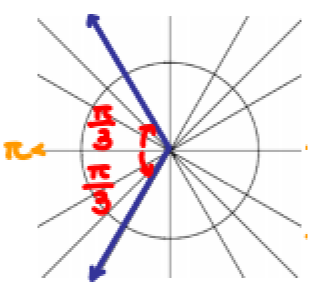
$\theta_1 = 30°\ \ and\ \ \theta_2 = 330°$



c) $\cos\theta = \frac{-1}{2} \qquad 0 \le \theta < 2\pi$

Cosine is negative in Q2 or Q3
The reference angle is $\frac{\pi}{3}$

$\theta_1 = \frac{2\pi}{3}\ \ and\ \ \theta_2 = \frac{4\pi}{3}$



d) $\csc\theta = 2 \qquad 0 \le \theta < 2\pi$

Cosecant is positive in Q1 or Q2
The reference angle is $\frac{\pi}{6}$

$\theta_1 = \frac{\pi}{6}\ \ and\ \ \theta_2 = \frac{5\pi}{6}$

Solving Trig Equations

Whereas solving an algebraic equation involves finding the value(s) of the variable, solving a trigonometric equation involves finding the value(s) of and angle (θ).

The solution(s) may need to be written in radians $(0 \le \theta \le 2\pi)$ or in degrees $(0 \le \theta \le 360°)$

Example 2: Determine the general solution in both degrees and radians. Give solutions as exact values.

a) $\sqrt{3}\tan\theta - 1 = 0$

$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\theta_1 = \frac{\pi}{6}$ $\quad \theta_3 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta_1 = \frac{\pi}{6} + 2\pi k$ $\quad \theta_2 = \frac{7\pi}{6} + 2\pi k$

$\theta_1 = 30° + 360°k$ $\quad \theta_2 = 210° + 360°k$

b) $-4 + 5\sin\theta = 4\sin\theta - 5$

$\sin\theta = -1$

$\theta = \frac{3\pi}{2}$ or $270°$

$\theta = \frac{3\pi}{2} + 2\pi k$

$\theta = 270° + 360°k$

Example 4: Solve and give solutions as exact values, where possible, in the given range.

c) $3\csc\theta - 6 = 0$ $\quad [-180°, 180°)$

$3\csc\theta = 6$

$\csc\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = 30°, 150°$

d) $\cos^2\theta - 3\cos\theta + 2 = 0$ $\quad -\pi \le \theta < \pi$

Let $u = \cos\theta$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2 \quad x = 1$

$\cos\theta = 2$ $\quad \cos\theta = 1$

no sols $\quad \theta = 0$

e) $\tan^2\theta - 5\tan\theta + 4 = 0$ $\quad [0, 2\pi)$

$(\tan\theta - 4)(\tan\theta - 1) = 0$

$\tan\theta = 4$ $\quad \tan\theta = 1$

$\theta_3 = \tan^{-1}(4)$ $\quad \theta_1 = \frac{\pi}{4}$

$= 1.326$ $\quad \theta_2 = \frac{5\pi}{4}$

$\theta_4 = 1.326 + \pi$

≈ 4.467

Example 3: Solve each equation on the interval $[0, 2\pi)$.

a) $\tan^2 x + \tan x = 0$

$$\tan x(\tan x + 1) = 0$$

$$\tan x = 0 \qquad \tan x = -1$$

$$x = 0, \pi \qquad x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

b) $2\sin^2 x - 5\sin x + 2 = 0$

$$(2\sin x - 1)(\sin x - 2) = 0$$

$$2\sin x = 1 \qquad \sin x = 2$$

$$\sin x = \frac{1}{2} \qquad \emptyset$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Example 4: Find **all** real solutions.

a) $2\sin^2 x - 1 = 0$

$$2\sin^2 x = 1$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm\frac{1}{\sqrt{2}}$$

$$x = \begin{cases} \frac{\pi}{4} + 2\pi k \\ \frac{3\pi}{4} + 2\pi k \\ \frac{5\pi}{4} + 2\pi k \\ \frac{7\pi}{4} + 2\pi k \end{cases}$$

b) $4\sin^2 x - 4\sin x + 1 = 0$

$$(2\sin x - 1)(2\sin x - 1) = 0$$

$$2\sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} + 2\pi k$$

$$x = \frac{5\pi}{6} + 2\pi k$$