Example 6: Combine the fractions and simplify to a multiple of a power of a basic trig function (e.g., $3\tan^2 x$)

b. $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} = \dfrac{\frac{1}{\cos x}}{\sin x} - \dfrac{\sin x}{\cos x}$

$\dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x} - \dfrac{\sin x}{\cos x}$

$\dfrac{1 - \sin x\left(\sin x\right)}{\cos x \sin x}$

$\dfrac{1 - \sin^2 x}{\cos x \sin x}$

$\dfrac{\cos^2 x}{\cos x \sin y} = \dfrac{\cos x}{\sin x} = \cot x$

Example 7: Rewriting a Trig Expression using Conjugates

a. $\dfrac{1}{1+\sin x}$

Solution

From the Pythagorean identity $\cos^2 x = 1 - \sin^2 x = (1 - \sin x)(1 + \sin x)$, you can see that multiplying both the numerator and the denominator by $(1 - \sin x)$ will produce a monomial denominator.

$\dfrac{1}{1+\sin x} = \dfrac{1}{1+\sin x} \cdot \dfrac{1-\sin x}{1-\sin x}$	Multiply numerator and denominator by $(1 - \sin x)$.
$= \dfrac{1-\sin x}{1-\sin^2 x}$	Multiply.
$= \dfrac{1-\sin x}{\cos^2 x}$	Pythagorean identity
$= \dfrac{1}{\cos^2 x} - \dfrac{\sin x}{\cos^2 x}$	Write as separate fractions.
$= \dfrac{1}{\cos^2 x} - \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\cos x}$	Product of fractions
$= \sec^2 x - \tan x \sec x$	Reciprocal and quotient identities

b. $\dfrac{5}{\tan x + \sec x} \cdot \dfrac{\tan x - \sec x}{\tan x - \sec x} = \dfrac{5\tan x - 5\sec x}{\tan^2 x - \sec^2 x} = \dfrac{5\tan x - 5\sec x}{-1}$

$= -5\tan x + 5\sec x$

Example 8: Trig Substitution

a. Use the substitution $x = 2\tan\theta, \quad 0 < \theta < \frac{\pi}{2}$ to write $\sqrt{4 + x^2}$ as a trig function of θ.

Solution

Begin by letting $x = 2\tan\theta$. Then, you can obtain

$\sqrt{4+x^2} = \sqrt{4+(2\tan\theta)^2}$	Substitute $2\tan\theta$ for x.
$= \sqrt{4+4\tan^2\theta}$	Rule of exponents
$= \sqrt{4(1+\tan^2\theta)}$	Factor.
$= \sqrt{4\sec^2\theta}$	Pythagorean identity
$= 2\sec\theta.$	$\sec\theta > 0$ for $0 < \theta < \pi/2$

b. Use the substitution $3x = 5\tan\theta, \quad 0 < \theta < \frac{\pi}{2}$ to write $\sqrt{9x^2 + 25}$ as a trig function of θ.

$$\sqrt{(3x)^2+25} = \sqrt{(5\tan\theta)^2+25} = \sqrt{25\tan^2\theta+25} = \sqrt{25(\tan^2\theta+1)}$$
$$= \sqrt{25\sec^2\theta}$$
$$= 5\sec^2\theta$$

Example 9: Rewriting a Logarithmic Expression

a. Rewrite $\ln|\csc\theta| + \ln|\tan\theta|$

Solution

$\ln\|\csc\theta\| + \ln\|\tan\theta\| = \ln\|\csc\theta\tan\theta\|$	Product Property of Logarithms
$= \ln\left\|\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}\right\|$	Reciprocal and quotient identities
$= \ln\left\|\frac{1}{\cos\theta}\right\|$	Simplify.
$= \ln\|\sec\theta\|$	Reciprocal identity

b. Rewrite $\ln|\sin\theta| + \ln|\cot\theta|$

$$\ln|\sin\theta\cot\theta| = \ln\left|\sin\theta\cdot\frac{\cos\theta}{\sin\theta}\right|$$
$$= \ln|\cos\theta|$$