4/24: QUOTIENT RULE

Rule #5: Quotient Rule

If $F(x) = \dfrac{f(x)}{g(x)}$, then $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$

Say the following: "Low Dee High minus High Dee Low, all over Low squared."

You have to say it with some rhythm. The "Dee" means derivative of. "Low" and "High" refer to the denominator and the numerator respectively. This is a much shorter sentence to remember, and feels much easier to say. This is something you would say to yourself, in your head, while you are taking a test. If you repeat this over and over, you won't mess up the formula when you need to use it!

Example 1: Find the derivative using two different derivative rules.

**IF you can simplify before you use the quotient rule, it may make the problem easier.

a) $f(x) = \dfrac{3x^{10} - 5x^8 + 12x^4 + 4x - 1}{2x}$

$= \frac{3}{2}x^9 - \frac{5}{2}x^7 + 6x^3 + 2 - \frac{1}{2}x^{-1}$

$f'(x) = \frac{27}{2}x^8 - \frac{35}{2}x^6 + 18x^2 + \frac{1}{2}x^{-2}$

$= \frac{27}{2}x^8 - \frac{35}{2}x^6 + 18x^2 + \frac{1}{2x^2}$

b) $f(x) = \dfrac{x^2 - 1}{x - 1} = \dfrac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$

$f(x) = x + 1$

$f'(x) = 1$

IF YOU CAN'T SPLIT OR FACTOR & REDUCE, THEN USE THE QUOTIENT RULE

Example 2: Find the derivative.

a) $y = \dfrac{2x}{x - 5}$

$y' = \dfrac{2(x-5) - 2x(1)}{(x-5)^2}$ QUOTIENT RULE

$= \dfrac{2x - 10 - 2x}{(x-5)^2}$ MULT OUT NUMERATOR

$= \dfrac{-10}{(x-5)^2}$ Combine Like Terms

b) $y=\dfrac{x^2+4x-5}{x-2}$

$$y'=\frac{(x-2)(2x+4)-1(x^2+4x-5)}{(x-2)^2}$$ Quotient Rule

$$=\frac{2x^2+4x-4x-8-x^2-4x+5}{(x-2)^2}$$ mult numerator

$$=\frac{x^2-4x-3}{(x-2)^2}$$ Combine like terms

c) $y=\dfrac{x^2-1}{(x-2)^3}$

$$y'=\frac{2x(x-2)^3-(x^2-1)\left[3(x-2)^2\cdot 1\right]}{(x-2)^6}$$ ~~Chain~~ Quotient Rule

$$=\frac{2x(x-2)^3-3(x^2-1)(x-2)^2}{(x-2)^6}$$ Put in order to factor

$$=\frac{\cancel{(x-2)^2}\left[2x(x-2)-3(x^2-1)\right]}{(x-2)^{\cancel{6}\,4}}$$ Reduce & Simplify [ ]

$$=\frac{2x^2-4x-3x^2+3}{(x-2)^4}=\frac{-x^2-4x+3}{(x-2)^4}$$ Combine Like Terms

d) $y=\dfrac{2x-3}{(3x-5)^4}$

$$y'=\frac{2(3x-5)^4-(2x-3)\left[4(3x-5)^3\cdot 3\right]}{(3x-5)^8}$$ Quotient Rule

$$=\frac{2(3x-5)^4-12(2x-3)(3x-5)^3}{(3x-5)^8}$$ Put in order to factor

$$=\frac{2\cancel{(3x-5)^3}\left[(3x-5)-6(2x-3)\right]}{(3x-5)^{\cancel{8}\,5}}$$ Factor out GCF, Reduce, Simplify [ ]

$$=\frac{2(3x-5-12x+18)}{(3x-5)^5}$$ Combine Like Terms

$$=\frac{2(-9x+13)}{(3x-5)^5}$$

Example 3: For a – d, write an expression for $f'(x)$ and then use it to find $f'(2)$ given the following $g'(-1) = 5, g(2) = 3, g'(2) = -2, h(2) = -1, \textit{and } h'(2) = 4$

a) $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$2(-2) + 4$

0

b) $f(x) = g(h(x))$

$f'(x) = g'(h(x)) \cdot h'(x)$

$g'(-1) \cdot 4$

$5 \cdot 4$

20

c) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + h(x)g'(x)$

$(3)(4) + (-1)(-2)$

14

d) $f(x) = \dfrac{h(x)}{g(x)}$

$f'(x) = \dfrac{g(x)h'(x) - h(x)g'(x)}{[g(x)]^2}$

$= \dfrac{(3)(4) - (-1)(-2)}{3^2} = \dfrac{10}{9}$

Example 4: The following table lists the values of functions *f* and *h*, and of their derivatives, *f'* and *h'*, for *x* = -3.

Find $\dfrac{d}{dx}\left[\dfrac{f(x)}{h(x)}\right]$ at x = -3

x	$f(x)$	$h(x)$	$f'(x)$	$h'(x)$
-3	8	-5	2	5

$\dfrac{h(x)f'(x) - f(x)h'(x)}{(h(x))^2}$

$\dfrac{(-5)(2) - (8)(5)}{25} = -\dfrac{50}{25} = -2$

Example 5: Evaluate $\dfrac{d}{dx}\left[\dfrac{f(x)}{h(x)}\right]$ at $x = 1$

$h(x) = \sqrt{x} = \sqrt{1} = 1$

$h'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2}(1)^{-\frac{1}{2}} = \frac{1}{2}$

- Let *f* be a function such that $f(1) = 0$ and $f'(1) = -7$.
- Let *h* be the function $h(x) = \sqrt{x}$ $\quad \frac{1}{2\sqrt{x}}$

$\dfrac{h(x)f'(x) - f(x)h'(x)}{(h(x))^2} = \dfrac{(1)(-7) - (0)(\frac{1}{2})}{1} = -7$

Example 6: Let $y(x) = \dfrac{z(x)}{1+x^2}$, $z(-3) = 6$, and $z'(x) = 15$. Find $y'(-3)$.

$y'(x) = \left.\dfrac{(1+x^2)z'(x) - z(x)\cdot 2x}{(1+x^2)^2}\right|_{x=-3} = \dfrac{(10)(15) - 6(-6)}{100}$

$= \dfrac{150 + 36}{100} = \dfrac{186}{100} = 1.86$

$= \dfrac{93}{50}$