4/20: PRODUCT RULE

> **Rule #4**: Product Rule
>
> If $F(x) = f(x) \cdot g(x)$, then $F'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$

For polynomial functions, it is not always necessary to use the product rule, however, with trigonometric, exponential, logarithmic, and other functions, it is a necessary tool.

Example 1: Find the derivative ~~first~~ without using the product rule, ~~and then with the product rule~~.
**Anything that can be distributed must be simplified.

a) $f(x) = (2x+3)(5x+7) = 10x^2 + 14x + 15x + 21$

$f(x) = 10x^2 + 29x + 21$

$f'(x) = 20x + 29$

b) $f(x) = (x^3 - 3x)(x^7 + 10)$

$f(x) = x^{10} + 10x^3 - 3x^8 - 30x$

$f'(x) = 10x^9 + 30x - 24x^7 - 30$

c) $f(x) = \sqrt[3]{x^2}\,(2x - x^2) = x^{2/3}(2x - x^2)$

$f(x) = 2x^{5/3} - x^{8/3}$

$f'(x) = \frac{10}{3}x^{2/3} - \frac{8}{3}x^{5/3}$

Example 2: Find the derivative.

a) $f(x) = 4x^2(7x-3)$

$f'(x) = 4x^2(7) + 8x(7x-3)$

$= 28x^2 + 56x^2 - 24x$

$f'(x) = 84x^2 - 24x$

b) $f(x) = (3x^2 - 2x + 4)(8x^3 + 4x^2 - 2)$

$f'(x) = (3x^2 - 2x + 4)(24x^2 + 8x) + (8x^3 + 4x^2 - 2)(6x-2)$

$= 72x^4 + 24x^3$
$- 2x^3 - 16x^2$
$+ 96x^2 + 32x$
$48x^4 - 16x^3$
$24x^3 - 8x^2 - 12x + 4$

$120x^4 + 30x^3 + 72x^2 + 20x + 4$

Example 3: Product Rule with Chain Rule

a) $f(x)=(2x+3)^3\,(5x+7)^4$

$f(x)=(2x+3)^3 \qquad f(x)'=3(2x+3)^2\cdot 2=6(2x+3)^2$

$g(x)=(5x+7)^4 \qquad g'(x)=4(5x+7)^3\cdot 5=20(5x+7)^3$

$$f'(x)=6(2x+3)^2\,(5x+7)^4+20(5x+7)^3\,(2x+3)^3$$

***Make sure to put coefficients at the front of each section. After that the parentheses order does not matter.

To finish this problem, we look for common factors and clean up the problem. Usually if the Chain Rule is involved, there will be a common factor.

$$\begin{aligned} f'(x)&=6(2x+3)^2\,(5x+7)^4+20(5x+7)^3\,(2x+3)^3\\ &=\underbrace{2(2x+3)^2\,(5x+7)^3}_{\textit{common factor}}\,\underbrace{[3(5x+7)+10(2x+3)]}_{\textit{simplify}} \end{aligned}$$

$$\begin{aligned} &=2(2x+3)^2\,(5x+7)^3\,[15x+21+20x+30]\\ &=2(2x+3)^2(5x+7)^3\,(35x+51) \end{aligned}$$

b) $f(x)=(6x-7)^4\,(7x-2)^5$

$f(x)=(6x-7)^4 \qquad f'(x)=4(6x-7)^3\cdot 6=24(6x-7)^3$

$g(x)=(7x-2)^5 \qquad g'(x)=5(7x-2)^4\cdot 7=35(7x-2)^4$

$$\begin{aligned} f'(x)&=24(6x-7)^3\,(7x-2)^5+35(7x-2)^4\,(6x-7)^4\\ &=\underbrace{(6x-7)^3\,(7x-2)^4}_{\textit{common factor}}\,\underbrace{[24(7x-2)+35(6x-7)]}_{\textit{simplify}} \end{aligned}$$

$$\begin{aligned} &=(6x-7)^3\,(7x-2)^4\,[168x-48+210x-245]\\ &=(6x-7)^3(7x-2)^4\,(378x-293) \end{aligned}$$

c) $y = x^2(x^3-2)^3$

$y' = x^2\left[3(x^3-2)^2(3x^2)\right] + 2x(x^3-2)^3$

$= 9x^4(x^3-2)^2 + 2x(x^3-2)^3$

$= x(x^3-2)^2\left[9x^3 + 2(x^3-2)\right]$

$x(x^3-2)^2(11x^3-4)$

Example 4: The following table lists the values of the functions of f and g, and of their derivatives f' and g', for $x = 1$. Evaluate $\frac{d}{dx}[f(x)\cdot g(x)]$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	7	−3

$f(x)g'(x) + f'(x)g(x)$

$3(-3) + 7(2)$

5

Example 5: Find $H'(4)$

- Let g be a function such that $g(4) = 8$ and $g'(4) = -3$.
- Let h be the function $h(x) = \sqrt{x}$ $\quad h(4) = \sqrt{4} = 2$
- Let H be a function defined as $H(x) = g(x)\cdot h(x)$

$h'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \qquad h'(4) = \frac{1}{4}$

$H'(4) = g(4)h'(4) + g'(4)h(4)$

$= 8\left(\frac{1}{4}\right) + -3(2)$

$= -4$

Closing: Find the derivative:

a) $f(x) = 5x^2(x+47)$

$f'(x) = 5x^2(1) + 10x(x+47)$

$= 5x^2 + 10x^2 + 470x$

$f'(x) = 15x^2 + 470x$

b) $f(x) = 5(x+47) = 5x + 235$

$f'(x) = 5$