Power and Constant Rule with Tangent to a Curve

Rule #1: Derivative of a Constant Function

If $f(x) = c$ then $f'(x) = 0$

This should not be too earth shattering to you, since the slope of a constant function is always 0!

Example 1: Find the derivative.

a. $f(x) = 6$

$f'(x) = 0$

b. $f(x) = -810$

$f'(x) = 0$

Rule #2: Power Rule

If $f(x) = x^n$ then $f'(x) = n \cdot x^{n-1}$

The use of the power rule is unchanged for *n* being a positive integer, *n* being a negative integer, or *n* being a rational number.

The KEY to using the power rule is to get comfortable using exponent rules to write a function as a power of x.

Example 2: Find the derivative.

a) $f(x) = x^5$ $\quad n=5$

$\boxed{f'(x) = 5x^4}$ $\quad n-1=4$

b. $f(x) = 3x^2$

$f'(x) = 6x^1$ or $6x$ ← move power to coeff & subt 1

c. $f(x) = \frac{1}{x}$ ← make problem be in correct form by make it have a power of −1

$= x^{-1}$

$f'(x) = -1x^{-2}$ ← Now that it is in power form, we can find f'

make sure to subt.

d. $f(x) = 2x^{-4}$ ← already in power form, so find f'

$f'(x) = -8x^{-5}$ ← Be sure to use correct notation

e. So what do we do if we have

$f(x) = 6x^3 - 4x^2 + 2x - 9$

To find $f'(x)$, take the derivative of each term

$f'(x) = 18x^2 - 8x + 2$ Remember, the derivative of a constant = 0

f. $f(x) = \frac{2x^2+5}{7}$ ⇐ we must separate this before finding f'

$f(x) = \frac{2}{7}x^2 + \frac{5}{7}$

$f'(x) = \frac{4}{7}x$ ⇐ Once we have it split, we can simplify if necessary & find f'

g. $f(x) = \frac{8x^3+5x^2-4x}{3x}$ ⇐ separate

$= \frac{8x^3}{3x} + \frac{5x^2}{3x} - \frac{4x}{3x}$ ⇐ reduce

$f(x) = \frac{8}{3}x^2 + \frac{5}{3}x - \frac{4}{3}$ ⇐ Now it is in power form, find f'

$f' = \frac{16}{3}x + \frac{5}{3}$ ⇐ make sure to reduce if necessary

Since the derivative is also the expression for the expression for the slope of the line tangent to the graph of a function, we can use the derivative to find the slope of the tangent at a given point.

Example 3: Find the slope of the line tangent. **To find the slope of the line tangent, we must find the derivative first, and then sub in the x-value into the derivative**

a) $f(x) = x^4 - 2x^3 + 5x^2 - 8$ at x = – 2

a) $f(x) = x^4 - 2x^3 + 5x^2 - 8$ AT $x = -2$

Find $f'(x) = 4x^3 - 6x^2 + 10x$

Now Evaluate $f'(-2)$ TO FIND THE SLOPE

$f'(-2) = 4(-2)^3 - 6(-2)^2 + 10(-2)$

$= \boxed{-76}$

∴ the slope at $x = -2$ is -76.

b) $f(x)=\frac{-3}{x^5}$ at x = – 1

b) $f(x) = \frac{-3}{x^5}$ AT $x=-1$ TO FIND $f'(x)$ we must change the Form first

$f(x) = -3x^{-5}$

$f'(x) = 15x^{-6}$ FIND f'

$f'(-1) = 15(-1)^{-6}$ Evaluate at $x=-1$

15 Slope is 15 AT $x=-1$

c) $f(x)=2x^4-3x^3+4x^2+2x+1$ at x = – 1

c. $f(x) = 2x^4 - 3x^3 + 4x^2 + 2x + 1$ AT $x=-1$

$f'(x) = 8x^3 - 9x^2 + 8x + 2$

$f'(-1) = 8(-1)^3 - 9(-1)^2 + 8(-1) + 2 = -23$

THE SLOPE AT $x=-1$ IS -23.

Once we are able to find the slope of the line tangent, we can write the equation of the line using the point-slope formula.

Ex 4: Find the equation of the line tangent at the given value. *Recall pt-slope formula:* $y - y_1 = m(x - x_1)$

a) $f(x)=3x^3-2x^2+4x-2$ at (2, 22) – called point of tangency

FIND $f'(x)$: $f'(x) = 9x^2 - 4x + 4$

FIND SLOPE AS IN Example 3

$f'(2) = 9(2)^2 - 4(2) + 4 = 32$

SO SLOPE IS 32

Plug into PT-SLOPE : $y - 22 = 32(x-2)$

$y - 22 = 32x - 64$ $+22$

$y = 32x - 42$

$\therefore$ $y = 32x - 42$ is the equation of the line tangent at $x=2$

b) $f(x) = 4x^4 - 2x^2 + 3$ at $x = -1$

b) $f(x) = 4x^4 - 2x^2 + 3$ AT $x = -1$

- Because we are writing the equation & not just finding slope, we will need the y-value at x = 1 since they did not give it to us this time.
- This is called the point of tangency, & to find y, we plug x into the original equation

Pt. of tangency : $(-1, \underline{5})$

$f(-1) = 4(-1)^4 - 2(-1)^2 + 3$

$f(-1) = 5$

Slope of line tangent

$f'(x) = 16x^3 - 4x$

Evaluate

$f'(-1) = 16(-1)^3 - 4(-1) = -12$

SO SLOPE OF TANGENT LINE IS -12

Equation of line tangent

$y - 5 = -12(x + 1)$

$y - 5 = -12x - 12$

$y = -12x + 7$

Remember, if you plug x into $f(x)$, you find y and the point of tangency. If you plug x into $f'(x)$, you find the slope of the line tangent.