4/3: RULES FOR DIFFERENTIATION

Basic Derivatives Rules

EXPONENT RULES

- NEGATIVE EXP: $a^{-x} = \frac{1}{a^x}$
- RATIONAL EXP: $\sqrt[b]{x^a} = x^{a/b}$

Constant Rule: If c is a constant, then $\frac{dy}{dx}[c] = 0$

Power Rule: $\frac{dy}{dx}[x^n] = nx^{n-1}$

Sum and Differences: $\frac{dy}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

$$\frac{dy}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

Remember last notes that the KEY to using the power rule is to get comfortable using exponent rules to write a function as a power of x.

Example 1: Find the derivative. WRITE WITH NO NEGATIVE EXPONENTS.

a) $f(x) = x^{\frac{1}{3}}$

b) $f(x) = \frac{1}{3}x^{-\frac{2}{3}}$

c) $f(x) = \sqrt[4]{x^3}$

d) $f(x) = \frac{7}{\sqrt{x^3}}$

e) $f(x) = \frac{8x^4 - 6x^3 + \pi}{x^2}$

f) $f(x) = 18x^{\frac{3}{2}} + 7x^{\frac{1}{3}}$

Many times it is helpful to use a little algebra BEFORE you try to use the derivatives rules…just use it correctly!

g) $g(x) = \frac{7}{(-2x)^4} - \frac{x}{2} + \frac{1}{4}$

h) $h(x) = (x^2 + 1)(2x - 5)$

EXAMPLE 1

a) $f(x) = x^{1/3}$ ALREADY A POWER

$f'(x) = \frac{1}{3}x^{-2/3}$ FIND f' USING POWER RULE

$f'(x) = \frac{1}{3x^{2/3}}$ HAS A NEG EXPONENT SO MOVE X TO DEN.

b) $f(x) = \frac{1}{3}x^{-2/3}$ USE POWER RULE

$f'(x) = -\frac{2}{9}x^{-5/3}$ Move x to den b/c its (–)

$f'(x) = \frac{-2}{9x^{5/3}}$

c) $f(x) = \sqrt[4]{x^3} = x^{3/4}$

- Change radical to power
- Now we can find f' using power rule
- Get rid of (–) exponent

$f'(x) = \frac{3}{4}x^{-1/4}$

$f'(x) = \frac{3}{4x^{1/4}}$

d) $f(x) = \frac{7}{\sqrt{x^3}} = \frac{7}{x^{3/2}}$

- Change to power

$f(x) = 7x^{-3/2}$

- Move x out of den to have a power

$f'(x) = -\frac{21}{2}x^{-5/2}$

- Find f'

$f'(x) = \frac{-21}{2x^{5/2}}$

- Get rid of (–) exponent

e) $f(x) = \dfrac{8x^4 - 6x^3 + \pi}{x^2}$

$f(x) = \dfrac{8x^4}{x^2} - \dfrac{6x^3}{x^2} + \dfrac{\pi}{x^2}$

$f(x) = 8x^2 - 6x + \pi x^{-2}$

$f'(x) = 16x - 6 - 2\pi x^{-3}$

$f'(x) = 16x - 6 - \dfrac{2\pi}{x^3}$

- SEPARATE INTO INDIVIDUAL TERMS & REDUCE
- NOW WE CAN FIND f'
- MOVE NEG POWER TO DEN OF TERM

f) $f(x) = 18x^{3/2} + 7x^{1/3}$

$f'(x) = 27x^{1/2} + \frac{7}{3}x^{-2/3}$

$f'(x) = 27x^{1/2} + \dfrac{7}{3x^{2/3}}$

- IN CORRECT FORM, USE POWER RULE
- TAKE CARE OF (−) EXP

g) $g(x) = \dfrac{7}{(-2x)^4} - \dfrac{x}{2} + \dfrac{1}{4}$

$g(x) = \dfrac{7}{16x^4} - \dfrac{x}{2} + \dfrac{1}{4}$

$g(x) = \frac{7}{16}x^{-4} - \frac{1}{2}x + \frac{1}{4}$

$g'(x) = -\frac{7}{4}x^{-5} - \frac{1}{2}$

$g'(x) = -\dfrac{7}{4x^5} - \dfrac{1}{2}$

- Simplify $(-2x)^4$ TO $16x^4$
- PUT IN POWER FORM
- FIND g'
- FIX (−) EXP

h) $h(x) = (x^2+1)(2x-5)$

$h(x) = 2x^3 - 5x^2 + 2x - 5$

$h'(x) = 6x^2 - 10x + 2$

- MUST MULTIPLY TO FIND INDIVIDUAL TERMS
- FIND h'

Example 2 $f(g(x)) = f'(g(x)) \cdot g'(x)$

a) $f(x) = (3x-7)^5$ Apply Chain Rule

$f'(x) = 5(3x-7)^4 \cdot (3)$ Mult 3·5

$f'(x) = 15(3x-7)^4$

b) $f(x) = 2(4x^3-3x^2+1)^3$

$f'(x) = 6(4x^3-3x^2+1)^2 \cdot (12x^2-6x)$ NO NEED TO MULT.

c) $f(x) = \sqrt{x+1}$ PUT IN POWER FORM

$f(x) = (x+1)^{1/2}$ Now we can apply Chain Rule

$f'(x) = \frac{1}{2}(x+1)^{-1/2} \cdot (1)$ CLEAN THIS UP B/C OF NEG EXPONENT

$f'(x) = \dfrac{1}{2(x+1)^{1/2}}$

d) $f(x) = \sqrt[5]{2x^4-1}$ PUT IN POWER FORM

$f(x) = (2x^4-1)^{1/5}$ Be-Careful not to put 4/5 as there is only 1 quantity

$f'(x) = \frac{1}{5}(2x^4-1)^{-4/5} \cdot (8x^3)$ Now use CHAIN RULE

$f'(x) = \dfrac{8x^3}{5(2x^4-1)^{4/5}}$ CLEAN UP

e) $f(x) = \frac{1}{\sqrt{x+4}}$

$f(x) = \frac{1}{(x+4)^{1/2}}$

$f(x) = (x+4)^{-1/2}$

PUT IN POWER FORM BY GETTING RID OF $\sqrt{}$ & MOVING VALUE TO NUMERATOR W/ NEG EXPONENT

$f'(x) = -\frac{1}{2}(x+4)^{-3/2} \cdot (1)$

APPLY CHAIN RULE

$f'(x) = -\frac{1}{2(x+4)^{3/2}}$

CLEAN UP INTO NO NEG EXP

Example 3 : REMEMBER TO FIND SLOPE, WE MUST FIND THE DERIVATIVE & SUB IN THE X-VALUE

$f(x) = (2x-7)^4$ AT $x = -2$

FIND f' USING CHAIN RULE

$f'(x) = 4(2x-7)^3 \cdot (2)$

$= 8(2x-7)^3$

SUB IN -2 FOR X

$f'(-2) = 8(2(-2)-7)^3$

$= -10648$

CALCULATE

THE SLOPE IS -10648 AT $x = -2$

Example 4: Recall from the last lesson, we need to find the slope and the point of tangency to write the equation.

a) Point of tangency at $x=-1$ (PLUG INTO $f(x)$) Calculate

$f(-1) = \left(4(-1)^4 - 2(-1)^2+3\right)^3$

$= 125$

Point is $(-1, 125)$

b) FIND f': $f(x) = (4x^4-2x^2+3)^3$ USE CHAIN RULE

$f'(x) = 3(4x^4-2x^2+3)^2 \cdot (16x^3-4x)$ SUB IN $x=-1$ TO FIND SLOPE

$f'(-1) = 3\left(4(-1)^4-2(-1)^2+3\right)^2 \cdot \left(16(-1)^3-4(-1)\right)$ Calculate

$m = {}^-900$

c) Write equation using Pt-Slope

$y-125 = -900\,(x+1)$

$y-125 = -900x-900$

$y = -900x-775$

Ex 5: Use the definitions of derivatives when not given an exact function

a) $\frac{dy}{dx}[f(x)-g(x)]$

$f'(x)-g'(x)$

$f'(3)-g'(3)$

$-3-(-5)$

$\boxed{2}$

• Use sum/diff rule

• GO TO TABLE AT $X=3$ & SUB IN VALUE OF TABLE

• EVALUATE

b) $\frac{dy}{dx}\left[\frac{1}{f(x)}\right] = \cancel{\frac{dy}{dx}[f^{-1}(x)]}$

$\frac{dy}{dx}[f(x)]^{-1}$

$-1[f(x)]^{-2}\cdot f'(x)$

$-1[f(3)]^{-2}\cdot f'(3)$

$-1(1)^{-2}\cdot(-3)$

$\boxed{3}$

• PUT IN PROPER FORM

• Apply Chain Rule

• SUB IN $X=3$

• GET VALUES FROM TABLE

• EVALUATE

c) $\frac{1}{\sqrt[3]{g(x)}} \Rightarrow [g(x)]^{-1/3}$

$\frac{dy}{dx}\left[[g(x)]^{-1/3}\right]$

$-1/3\,[g(x)]^{-4/3}\cdot g'(x)$

$-1/3\,[g(3)]^{-4/3}\cdot g'(3)$

$-1/3\,[8]^{-4/3}\cdot(-5)$

$\boxed{\frac{5}{48}}$

• Rewrite into proper form

• Use CHAIN RULE

• SUB IN $X=3$

• FIND VALUES IN CHART

• Calculate

d) $\sqrt{g(x)} \Rightarrow [g(x)]^{+1/2}$

$\Rightarrow +1/2\,[g(x)]^{-1/2} \cdot g'(x)$

$\Rightarrow \frac{1}{2}\,[8]^{-1/2} \cdot (-5)$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{8}} \cdot \frac{-5}{1}$

$= \frac{1}{2} \cdot \frac{1}{2\sqrt{2}} \cdot \frac{-5}{1}$

$= \frac{-5}{4\sqrt{2}}$

e) $f(f(x)) \Rightarrow f'(f(x)) \cdot f'(x)$ Use Chain Rule Def

$\Rightarrow f'(f(3)) \cdot f'(3)$

$= f'(1) \cdot (-3)$ FIND Values IN table

$= 5 \cdot (-3)$

$= \boxed{-15}$

f) $g(g(x)) \Rightarrow g'(g(x)) \cdot g'(x)$ Use Chain Rule Def

$\Rightarrow g'(g(3)) \cdot g'(3)$

$\Rightarrow g'(8) \cdot -5$

$\Rightarrow 4 \cdot -5$

$= -20$