Extreme Value Notes

I. Prerequisite Skills: Recall that to find the x-intercepts, we set the function = 0 & solve

a. $8x^2 + 10x + 3 = 0$

$8x^2 - 6x - 4x + 3 = 0$

$2x(4x-3) - 1(4x-3) = 0$

$(2x-1)(4x-3) = 0$

$x = 1/2 \quad x = 3/4$

$\therefore$ xint $(\frac{1}{2}, 0)$ $(3/4, 0)$

b. $x^4 - 5x^2 - 36 = 0$

$x^4 - 9x^2 + 4x^2 - 36 = 0$

$x^2(x^2-9) + 4(x^2-9) = 0$

$(x^2+4)(x^2-9) = 0$

$x = \pm 2i \quad x = \pm 3$

$\therefore$ xint $(\pm 3, 0)$

II. First Derivative Test for Critical Pts

Def: Let f be defined at c. If $f'(c) = 0$ or f is not differentiable at c, then c is a critical number of f and the point $(c, f(c))$ is a critical point of f.

Ex 1: To find the critical pts, we must first find f' and then solve the derivative by setting it = 0.

a. $f(x) = 3x^3 - 9x + 5$

$f'(x) = 9x^2 - 9$ ← Find f' using Power Rule

$9x^2 - 9 = 0$ ← Set $f' = 0$

$9(x^2 - 1) = 0$ ← Choose a Quad solving method

$9(x-1)(x+1) = 0$

$x = 1 \quad x = -1$

$\therefore$ $x = 1$ & $x = -1$ are critical numbers

b) $f(x) = 2x^3 - 15x^2 + 24x + 7$

$f'(x) = 6x^2 - 30x + 24$ — FIND f'

$6(x^2 - 5x + 4) = 0$ — SET $= 0$

$6(x - 1)(x - 4) = 0$ — FACTOR

$\boxed{x = 1 \quad x = 4}$ — CRIT. #S

c) $f(x) = 3x^3 - 18x^2 - 4$

$f'(x) = 9x^2 - 36x$ — FIND f'

$9x^2 - 36x = 0$ — SET $= 0$

$9x(x - 4) = 0$ — Factor USING GCF

$\boxed{x = 0 \quad x = 4}$ — Crit #s

TEST FOR INC / DEC FUNCTIONS

1) IF $f'(x) > 0$ FOR ALL x IN (a, b), then $f(x)$ IS INCREASING ON $[a, b]$

2) IF $f'(x) < 0$ FOR ALL x IN (a, b), then $f(x)$ IS DECREASING ON $[a, b]$

3. IF $f'(x) = 0$ FOR ALL x IN (a, b), then $f(x)$ IS CONSTANT ON $[a, b]$

FIRST DERIVATIVE TEST FOR CRITICAL PTS: ASSUME THAT c IS A CRITICAL NUMBER OF A FUNCTION, $f(x)$. THEN

- IF $f'(x)$ CHANGES FROM POSTIVE TO NEGATIVE AT c, THEN $(c, f(c))$ IS A RELATIVE MAX OF f.

- IF $f'(x)$ CHANGES FROM NEGATIVE TO POSITIVE AT c, THEN $(c, f(c)$ IS A RELATIVE MINIMUM OF f.

Steps to Finding Intervals of Inc / Dec

1) Find the critical numbers of f and use them to create test intervals (think sign chart)
2) Choose a number w/in each interval to test
3) Use Test for Inc / Dec on each interval

Examples

a) $f(x) = x^3 - 27x - 20$

$f'(x) = 3x^2 - 27$ ← Find f'

$3(x^2 - 9) = 0$ ← Solve

$3(x-3)(x+3) = 0$

$x = 3 \quad x = -3$

$-\,-\ P \quad | \quad -\,+\ N \quad | \quad +\,+\ P$ ← Number line for test

$\nearrow \quad -3 \quad \searrow \quad 3 \quad \nearrow$

Sub in here for sign chart

The pos int inc & neg ones dec

Inc $(-\infty, -3) \cup (3, \infty)$

Dec $(-3, 3)$

$x = -3$ max $\quad x = 3$ min

← Int go from inc/dec or vice versa

b) $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

$3x(x-1) = 0$

$x = 0 \quad x = 1$ ← crit pts

$-\,-\ P \quad | \quad +\,-\ N \quad | \quad +\,+\ P$ ← Is $f' > 0$ or $f' < 0$

$\nearrow \quad 0 \quad \searrow \quad 1 \quad \nearrow$

Inc $(-\infty, 0) \cup (1, \infty)$

Dec $(0, 1)$

$x = 0$ max

* Remember on the sign chart, we don't care what the value is, we just want to know if it is pos or negative

* If $f' > 0$, then inc
  If $f' < 0$, then dec

c) $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$

$g'(x) = x^2 + 3x - 70 = 0$

$= (x-7)(x+10)$

$x = 7 \quad x = -10$

$$\begin{array}{ccccc} -\,-\;P & & -\,+\;N & & +\,+\;P \\ \nearrow & -10 & \searrow & 7 & \nearrow \end{array}$$

INC : $(-\infty, -10) \cup (7, \infty)$

DEC : $(-10, 7)$

$x = -10$ MAX

$x = 7$ MIN

d) $f(x) = \frac{1}{3}x^3 - x^2 + x$

$f'(x) = x^2 - 2x + 1$

$= (x-1)(x-1)$

$x = 1$ (mult 2)

$$\begin{array}{ccc} P & & P \\ \nearrow & 1 & \nearrow \end{array}$$

INC : $(-\infty, 1) \cup (1, \infty)$ 1 IS NOT INCLUDED BECAUSE ITS VALUE IS 0

NO DEC

NO MAX OR MIN SINCE f' IS NOT CHANGING FROM POS TO NEG OR VICE VERSA

III. Concavity & Pts of Inflection

To determine Concavity

1) Find f'' & set $= 0$
2) Create sign chart & sub into f'' to test
3) If $f''(x) > 0$, the interval is concave up
   If $f''(x) < 0$, the interval is concave down

a) $f(x) = 3x^5 - 5x^4 + 1$

$f'(x) = 15x^4 - 20x^3$

$f''(x) = 60x^3 - 60x^2 = 0$ Find f''

$60x^2(x-1) = 0$ Set $= 0$ & solve

$x = 0$ $x = 1$

$+-$ N $+-$ N $++$ P

$\cap$ 0 $\cap$ 1 $\cup$

Test intervals by subbing into f''

Concave Down: $(-\infty, 0) \cup (0, 1)$

Concave Up: $(1, \infty)$

$x = 1$ is a pt of inflection since f'' changes from con down to con up

b) $f(x) = x^4 - 4x^3$

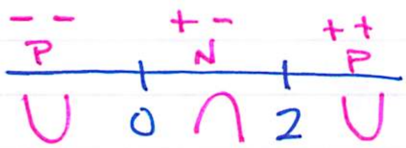
$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$x = 0 \qquad x = 2$

FIND f''
SET = 0 & SOLVE



PLACE ON SIGN CHART & SUB INTO FACTORED FORM OF f''

Concave Up $(f''(x) > 0)$: $(-\infty, 0) \cup (2, \infty)$

Concave Down $(f''(x) < 0)$: $(0, 2)$

$x = 0$ & $x = 2$ are points of inflection